

Incentive Mechanism Design for Heterogeneous Peer-to-Peer Networks: A Stackelberg Game Approach

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Abstract

With high scalability, high video streaming quality, and low bandwidth requirement, peer-to-peer (P2P) systems have become a popular way to exchange files and deliver multimedia content over the internet. However, current P2P systems are suffering from “free-riding” due to the peers’ selfish nature. In this paper, we propose a credit-based incentive mechanism to encourage peers to cooperate with each other in a heterogeneous network consisting of wired and wireless peers. The proposed mechanism can provide differentiated service to peers with different credits through biased resource allocation. A Stackelberg game is formulated to obtain the optimal pricing and purchasing strategies, which can jointly maximize the revenue of the uploader and the utilities of the downloaders. In particular, peers’ heterogeneity and selfish nature are taken into consideration when designing the utility functions for the Stackelberg game. It is shown that the proposed resource allocation scheme is effective in providing service differentiation for peers and stimulating them to make contribution to the P2P streaming system.

Index Terms

Game Theory, Stackelberg Game, Network Optimization, Credit-based Incentive Mechanism, Peer-to-Peer Networks, Heterogeneous Networks.

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I. INTRODUCTION

With the rapid development of peer-to-peer (P2P) communication technologies, P2P networks have become a popular way to exchange files and deliver multimedia content over the internet due to their low bandwidth requirement, good video streaming quality, and high flexibility. However, current P2P systems greatly rely on volitional resource contribution from individual peers and do not enforce any compulsory contribution from these peers. This directly leads to the well-known “free-riding” problem, which refers to the phenomenon that a peer consumes free service provided by other peers without contributing any its resources to the P2P network. This tremendously degrades the performance of P2P systems, especially P2P multimedia streaming systems which have high requirements on time delay and data rate. Free-riding is common in P2P networks due to peers’ selfish nature and the limited network resources. Most peers only want to maximize their own benefits without caring about the overall performance of the whole P2P community. It is reported in [2] that more than 70% P2P users do not share any file in Gnutella system. Therefore, to enhance the performance of P2P networks, effective incentive mechanisms need to be put in place to stimulate the cooperation between peers and encourage them to make contribution to the P2P system.

On the other hand, recent advances in wireless communications technologies (3G/4G networks) and smart phones have enabled the development of mobile version of P2P applications for smart phones, such as PPtv [3] and PPStream [4]. People use these mobile P2P applications to watch movies, watch dramas, or listen to music when traveling on buses and metros. Due to the convenience, mobile P2P users are increasing dramatically nowadays. As compared to the wired P2P users, mobile P2P users are more selfish due to the high cost of mobile data. Thus, there is also a compelling need to design effective incentive mechanisms for mobile P2P applications.

The existing incentive mechanisms for P2P systems are mainly designed to work in wired networks. For the heterogeneous networks with both wired and wireless nodes, these incentive mechanisms may not work well due to the differences between the wired nodes and the wireless nodes. For example, the computing capability of the wireless nodes (such as smart phones and tablet PCs) is usually weaker than that of the wired nodes (such as desktop PCs, and workstations). Thus, incentive mechanisms with high complexity may not be suitable for mobile applications. It is true that there exist high-end smartphones with high-end four-core or eight-

core processors. However, incentive mechanisms with high complexity are still not preferred on these mobile devices since the high complexity computing can drain out the devices' batteries fast. In addition, the connection bandwidth of the wireless nodes is usually less than that of the wired nodes. This should be taken into consideration when designing the incentive mechanism to achieve relative fairness. However, to the best of our knowledge, most of the existing work fails to do this. All these differences between the wireless and wired nodes pose new challenges to the design of the incentive mechanism for the heterogeneous networks.

In this paper, we propose a credit-based incentive mechanism for heterogeneous networks with both wired and wireless nodes. We consider a P2P streaming network where each peer can serve as an uploader and a downloader at the same time. When a peer uploads data chunks to other peers, it can earn certain credits for providing the service. When a peer downloads data chunks from other peers, it has to pay certain credits for consuming the resource. A peer's net contribution to the network is reflected by its accumulated credits. A Stackelberg game is formulated to provide differentiated service to peers with different credits. Particularly, peers' heterogeneity and selfish nature are taken into consideration when designing the utility functions.

The main contributions and key results of this paper are summarized as follows.

- A credit-based incentive mechanism based on Stackelberg games is proposed for P2P streaming networks. To the best of our knowledge, this is the first work that applies the Stackelberg game to the incentive mechanism design for P2P streaming networks.
- Peers' heterogeneity is taken into consideration when designing the utility functions for the Stackelberg game. Thus, our incentive mechanism can be applied to heterogeneous P2P networks with wired and wireless peers having different connection bandwidths.
- The selfish nature of peers is taken into consideration when designing the utility functions for the Stackelberg game, i.e., every peer is a strategic player with the aim to maximize its own benefit. This makes our incentive mechanism perform well in a P2P network environment with non-altruistic peers.
- The optimal pricing strategies for the uploader and the optimal purchasing strategies for the downloader are both derived. The Stackelberg equilibrium is then obtained and shown to be unique and Pareto-optimal.
- Two fully distributed implementation schemes are proposed based on the obtained theoretical results. It is shown that each of these schemes has its own advantages.

- The impact of peer churn on the proposed incentive mechanism is analyzed. It is shown that the proposed mechanism can adapt to dynamic events such as peers joining or leaving the network.

The remaining parts of this paper are organized as follows. In Sections II and III, we present the related work and describe our system model. In Sections IV and V, we present the problem formulation and its optimal solution. In Sections VI and VII, we propose two implementation schemes and study the impact of peer churn on the proposed schemes. Numerical results are given in Section VIII to evaluate the performance of the proposed schemes. Then, we discuss some possible extensions of this work in Section IX. Section X concludes the paper.

II. RELATED WORK

A simple incentive mechanism for P2P systems is the “tit-for-tat” strategy, where peers receive only as much as they contribute. A free rider that does not upload data chunks to other peers cannot get data chunks from them and suffers from poor streaming quality. Due to its simplicity and fairness, this scheme has been adopted by BitTorrent [5]. Though this strategy can increase the cooperation between peers to a certain level, it is shown in literature [6]–[8] that it may perform poorly in today’s internet environment due to the asymmetry of the upload and download bandwidths.

Unlike the “tit-for-tat” strategy, which enforces compulsory contribution from peers, another category of incentive mechanisms stimulate peers to contribute to the system by indirect reciprocity [9]–[17]. In these incentive mechanisms, the contribution of each peer is converted to a score which is then used to determine the reputation or rank of the peer among all the peers in the network. Peers with a high reputation are given a certain priority in utilizing the network resources, such as selecting peers or desirable media data chunks. Therefore, peers with a high reputation have more flexibility in choosing desired data suppliers and thus are more likely to receive high-quality streaming. On the other hand, peers with a low reputation have quite limited options in parent-selection and thus receive low-quality streaming. Through this way, the P2P systems can provide differentiated service to peers with different reputation values. Hence, peers are motivated to contribute more to the P2P system to earn a higher reputation.

Recently, game theory [18] is found to be a powerful tool to study strategic interactions among rational peers and design incentive mechanisms to stimulate the cooperation among peers for

P2P streaming systems. This is due to the fact that peers are selfish and strategic players in P2P streaming systems. It is their inherent nature to maximize their payoffs while simultaneously reducing their cost, i.e., enjoying a high quality streaming service while consuming least of their own resource. Game theory has been widely used in studying strategic interactions among these peers [19]–[29]. In [19]–[23], the authors discussed how to apply game theory to the design of incentive mechanisms for P2P networks at a high level. It is pointed out that straightforward use of results from traditional game theory do not fit well with the requirements of P2P networks. The utility functions must be customized for P2P networks. In [24], a repeated static game called Cournot Oligopoly game was formulated to model the interactions between peers, and an incentive mechanism was proposed by analyzing and solving the game. In [25], a simple, selfish, link-based incentive mechanism for unstructured P2P file sharing systems was proposed. It was shown that a greedy approach is sufficient for the system to evolve into a “good” state under the studied game model. In [26], an incentive mechanism was proposed for P2P networks based on the Bayes game. In [27], an infinitely repeated game was formulated to analyze the interactions between peers, and a so-called *credit line* mechanism was proposed to stimulate cooperation between peers. In [28], based on the *first-price auction* procedure, a payment-based incentive mechanism was proposed for P2P streaming networks. Whereas, in [29], a non-cooperative competition game was used to provide service-differentiated resource allocation between competing peers in a P2P network.

Different from these work, to the best of our knowledge, our work is the first work that models the peers’ interactions as a Stackelberg game. Particularly, we take the peers’ heterogeneity (wired/wireless peers with different connection bandwidths) into consideration when designing the utility functions for the Stackelberg game. Besides, two distributed implementations of the mechanism with different complexity are proposed to handle the difference in computing capability between wired and wireless nodes.

III. SYSTEM MODEL

In this paper, we consider a P2P streaming network where all the peers can serve as the uploader and the downloader at the same time. To eliminate the free-riding phenomenon and encourage cooperation between peers, we introduce the concept of credit into the system, where peers earn credits for providing service and consume credits for receiving service. We assume

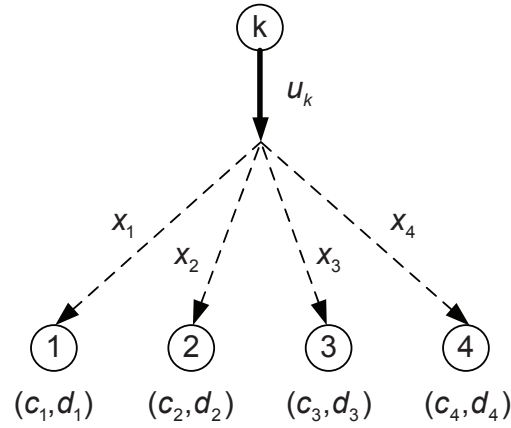


Fig. 1. System Model

that all the peers are selfish and rational. Their aim is to maximize the credits that they can earn by fully utilizing their available network resource. Each peer has the right to set up a price for the service that it provides based on its own benefits. For fairness considerations, we assume that the uploader can only adopt the uniform pricing strategy, i.e., it cannot set different prices for different peers for the same amount of bandwidth allocation.

In this paper, the credit of peer i is denoted by c_i . The connection type (i.e., the download capacity) of peer i is denoted by d_i . The download bandwidth allocation for peer i is denoted by x_i . The upload bandwidth of peer k is denoted by u_k . We denote the set of peers that request data chunks from peer k as S_k . To avoid trivial bandwidth allocation schemes, we assume that $\sum_{i \in S_k} d_i > u_k$. As illustrated in Fig. 1, the downloaders send their credits and connection types to the the uploader together with their data request. The uploader then decides the bandwidth price and allocates the bandwidth to requesters based on their credits and connection types. For example, suppose there are 100 peers requesting data chunks from peer i , but peer i can only provide service to 20 peers at the same time due to its limited upload bandwidth. Then, peer i can set up a high price that only 20 peers can accept, and the remaining 80 peers will give up due to the high cost. Through this way, peers with more credits are actually given a higher priority in utilizing network resources, and service differentiation for peers with different credits is thus realized. In this paper, the above service differentiation scheme is realized by a Stackelberg resource allocation game which is investigated in the following section.

IV. STACKELBERG GAME FORMULATION

A Stackelberg game is a strategic game that consists of a leader and several followers competing with each other on certain resources. The leader moves first and the followers move subsequently. In this paper, we formulate the uploader that has media data chunks as the leader, and the downloaders that request for media data chunks as the followers. The uploader (leader) imposes a price on each unit of bandwidth providing to each downloader. Then, the downloaders (followers) determine their optimal download bandwidths to maximize their individual utilities based on the assigned bandwidth price. The Stackelberg Game consists of two parts: the uploading game at the uploader side and the downloading game at the downloader side, which are introduced in the following two subsections, respectively.

A. Uploading Game Design

Under the Stackelberg game model, for the uploader k , if we denote its price on each unit of bandwidth providing to each downloader as μ , then its revenue maximization problem can be formulated as

Uploading Game:

$$\max_{\mu > 0} \sum_{i \in S_k} \mu x_i, \quad (1)$$

$$\text{s.t.} \quad \sum_{i \in S_k} x_i \leq u_k, \quad (2)$$

where x_i is the bandwidth that peer i intends to purchase, and x_i is a function of the bandwidth price μ , i.e., $x_i \triangleq f_i(\mu)$. S_k denotes the set of peers that request data chunks from peer k , and u_k is the total available upload bandwidth of peer k .

Under the Stackelberg game formulation, the amount of bandwidth that peer i intends to purchase is decreasing in the bandwidth price μ . On the other hand, it is observed from (1) that the revenue of the uploader is the sum of products of the bandwidth price and individual peer's purchased bandwidth. Therefore, the uploader must carefully design its bandwidth pricing strategy in order to maximize its revenue.

B. Downloading Game Design

At the downloader side, for each peer i that requests data chunks from the uploader, the utility maximization problem can be formulated as

Downloading Game:

$$\max_{x_i \geq 0} c_i s_i - \mu x_i, \quad (3)$$

$$\text{s.t. } x_i \leq d_i, \quad (4)$$

where $s_i \triangleq \log_2 \left(1 + \frac{x_i}{d_i} \right)$ is the *performance satisfaction factor* for peer i , c_i is the credits that peer i has, and d_i is the maximum download bandwidth of peer i .

The performance satisfaction factor s_i reflects the degree of satisfaction or the “happines” of the downloader under the received bandwidth x_i . A log function is adopted to model this factor due to the fact that log functions are shown in literature to be suitable to representing a large class of elastic data traffics including the media streaming service [30], [31]. When the received bandwidth $x_i = 0$, the satisfaction factor s_i is equal to 0, which indicates that peer i is unsatisfied with its system performance. On the other hand, when the received bandwidth $x_i = d_i$, the satisfaction factor s_i is equal to 1, which indicates that peer i is fully satisfied with its system performance. The degree of satisfaction increases with the increase of the received bandwidth x_i .

It is also observed from (3) that the utility function of the downloader consist of two parts: $c_i s_i$ and μx_i . $c_i s_i$ is the credits that peer i is willing to pay for the service it received, while μx_i is the cost that peer i has to pay for obtaining the bandwidth x_i . Obviously, with a larger bandwidth x_i , peer i can obtain more satisfactory system performance, and thus is willing to pay more credits. On the other hand, the cost increases with the increase of the bandwidth x_i . Therefore, optimal strategies are needed for a rational peer to balance its cost and the achieved system performance in order to maximize its utility.

C. Stackelberg Equilibrium

The uploading game and the downloading game together form a Stackelberg game. The objective of this game is to find the Stackelberg Equilibrium (SE) point(s) from which neither the leader nor the followers have incentives to deviate. For the proposed Stackelberg game, the SE is defined as follows.

Definition 3.1: Let μ^* be a solution for the uploading problem and x_i^* be a solution for the downloading game of the i th peer. Then, the point (μ^*, \mathbf{x}^*) is a SE for the proposed Stackelberg

game if for any (μ, \mathbf{x}) with $\mu > 0$ and $\mathbf{x} \succeq \mathbf{0}$, the following conditions are satisfied:

$$U^{up}(\mu^*, \mathbf{x}^*) \geq U^{up}(\mu, \mathbf{x}^*), \quad (5)$$

$$U_i^{down}(x_i^*, \mu^*) \geq U_i^{down}(x_i, \mu^*), \forall i, \quad (6)$$

where $U^{up}(\cdot)$ and $U^{down}(\cdot)$ are the utilities of the uploading game and the downloading game, respectively.

For the proposed game in this paper, the SE can be obtained as follows: For a given price μ , the downloading game is solved first. Then, with the obtained best response functions \mathbf{x}^* of the downloaders, we solve the uploading game for the optimal price μ^* .

V. OPTIMAL RESOURCE ALLOCATION STRATEGIES

In this section, we investigate the optimal resource allocation strategies for the proposed Stackelberg game, i.e., the optimal bandwidth allocation for the downloading game and the optimal pricing strategy for the uploading game.

A. Optimal Download Bandwidth

For a given μ , the optimal bandwidth x_i^* for peer i is given in the following theorem.

Theorem 4.1: For a given μ , the optimal solution for the downloading game is

$$x_i^* = \begin{cases} d_i, & \text{if } \mu \leq \frac{c_i}{2d_i \ln 2}, \\ \frac{c_i}{\mu \ln 2} - d_i, & \text{if } \frac{c_i}{2d_i \ln 2} < \mu \leq \frac{c_i}{d_i \ln 2}, \\ 0, & \text{if } \mu > \frac{c_i}{d_i \ln 2}. \end{cases} \quad (7)$$

Proof: The Lagrangian of the downloading game can be written as

$$L(x_i, \alpha, \beta) = c_i \log_2 \left(1 + \frac{x_i}{d_i} \right) - \mu x_i - \alpha (x_i - d_i) + \beta x_i.$$

where α and β are the nonnegative dual variable associated with the constraints.

The dual function is $q(\alpha, \beta) = \max_{x_i} L(x_i, \alpha, \beta)$. The Lagrange dual problem is then given by $\min_{\alpha \geq 0, \beta \geq 0} q(\alpha, \beta)$. The duality gap is zero for the convex problem addressed here, and thus solving its dual problem is equivalent to solving the original problem. Thus, the optimal solutions

needs to satisfy the following Karush-Kuhn-Tucker (KKT) conditions [32]:

$$d_i \geq x_i^* \geq 0, \alpha \geq 0, \beta \geq 0, \quad (8)$$

$$\alpha (x_i^* - d_i) = 0, \quad (9)$$

$$\beta x_i^* = 0, \quad (10)$$

$$\frac{\partial L(x_i^*, \alpha, \beta)}{\partial x_i^*} = \frac{c_i}{(d_i + x_i^*) \ln 2} - \mu - \alpha + \beta = 0. \quad (11)$$

From (11), it follows

$$x_i^* = \frac{c_i}{(\mu + \alpha - \beta) \ln 2} - d_i. \quad (12)$$

Suppose $x_i^* < d_i$ when $\mu \leq \frac{c_i}{2d_i \ln 2}$. Then, from (9), it follows $\alpha = 0$. Therefore, (12) reduces to $x_i^* = \frac{c_i}{\ln 2(\mu - \beta)} - d_i$. Then $x_i^* < d_i$ results in $\frac{c_i}{2d_i \ln 2} < \mu - \beta$. Since $\beta \geq 0$, it follows $\mu > \frac{c_i}{2d_i \ln 2}$. This contradicts the presumption. Therefore, from (9), it follows

$$x_i^* = d_i, \text{ if } \mu \leq \frac{c_i}{2d_i \ln 2}. \quad (13)$$

Similarly, we can prove that $x_i^* = 0$, if $\mu > \frac{c_i}{d_i \ln 2}$, and $x_i^* = \frac{c_i}{\mu \ln 2} - d_i$, if $\frac{c_i}{2d_i \ln 2} < \mu \leq \frac{c_i}{d_i \ln 2}$.

Theorem 4.1 is thus proved. ■

Remark: It is observed from (7) that x_i^* is a piecewise function of the price μ . If the price μ is very high, the optimal download bandwidth x_i^* for peer i is 0; if the price μ is very low, peer i will download at its maximum bandwidth. In general, x_i^* is a decreasing function of μ . This indicates that the uploader can easily control the bandwidth allocated to peer i by controlling the price μ assigned for peer i . Besides, some key observations obtained from (7) are listed as follows.

- Under the same prescribed price μ , comparing with the same type (i.e., the same d_i) of peers with higher contributions (i.e., more credits), low contributors are more likely to be rejected from downloading. The uploader can easily reject a low contributor i from downloading by setting a price larger than $\frac{c_i}{d_i \ln 2}$.
- Under the same prescribed price μ , more bandwidth is allocated to the peer with higher contributions for the same type of peer. High contributors are more likely to download at their maximum download bandwidth.

B. Optimal Pricing Strategy

With the results obtained in (7), we are now ready for solving the uploading game. Using $f_i(\mu)$ to denote the x_i obtained in (7), the uploading game can be rewritten as

$$\text{P1: } \max_{\mu > 0} \sum_{i \in S_k} \mu f_i(\mu), \quad (14)$$

$$\text{s.t. } \sum_{i \in S_k} f_i(\mu) \leq u_k. \quad (15)$$

The above problem is difficult to solve since $f_i(\mu)$ is a piece-wise function of μ . Therefore, to solve P1, we first consider the two-peer scenario, and then extend the results to the multi-peer scenario.

1) *Two-peer Scenario*: In this scenario, we consider the case that only two peers request data chunks from the uploader, and we assume that they are sorted in the order $\frac{c_1}{d_1} > \frac{c_2}{d_2}$. Then, the thresholds given in (7) of the two peers may have the following two possible orders.

$$\text{Case I: } \frac{c_1}{d_1 \ln 2} > \frac{c_2}{d_2 \ln 2} > \frac{c_1}{2d_1 \ln 2} > \frac{c_2}{2d_2 \ln 2}.$$

$$\text{Case II: } \frac{c_1}{d_1 \ln 2} > \frac{c_1}{2d_1 \ln 2} > \frac{c_2}{d_2 \ln 2} > \frac{c_2}{2d_2 \ln 2}.$$

Before we start the analysis for the above two cases, the upper limit and the lower limit of the optimal price μ^* are given out in the following two propositions.

Proposition 4.1: The upper bound of μ^* is $\frac{c_1}{d_1 \ln 2}$, i.e., $\text{Sup}(\mu^*) = \frac{c_1}{d_1 \ln 2}$.

Proof: This can be proved by contradiction. Suppose the optimal μ^* of P1 satisfies $\mu^* \geq \frac{c_1}{d_1 \ln 2}$. Then, it follows that $\mu^* \geq \frac{c_2}{d_2 \ln 2}$ since $\frac{c_1}{d_1} > \frac{c_2}{d_2}$. According to (7), we have $x_1 = 0$ and $x_2 = 0$. The resulting revenue for the uploader is zero. It is easy to see that we can find another pricing strategy μ' that satisfies $\mu' < \frac{c_1}{d_1 \ln 2}$ and can generate a revenue larger than zero. This contradicts with our presumption. Thus, $\mu^* \geq \frac{c_1}{d_1 \ln 2}$ does not hold. Therefore, μ^* must be less than $\frac{c_1}{d_1 \ln 2}$. ■

Proposition 4.2: The minimum value for μ^* is $\frac{c_2}{2d_2 \ln 2}$, i.e., $\text{Min}(\mu^*) = \frac{c_2}{2d_2 \ln 2}$.

Proof: It is observed from (7) that $x_1 = d_1$ and $x_2 = d_2$ if $\mu = \frac{c_2}{2d_2 \ln 2}$. It is clear that x_1 and x_2 will not increase if the uploader sets a lower μ , which indicates that the uploader cannot increase its revenue by setting a price lower than $\frac{c_2}{2d_2 \ln 2}$. Therefore, the minimum value for the optimal price μ^* is $\frac{c_2}{2d_2 \ln 2}$. ■

Now, we solve P1 for the two-peer scenario under the above two cases, respectively. First, we consider Case I: $\frac{c_1}{d_1 \ln 2} > \frac{c_2}{d_2 \ln 2} > \frac{c_1}{2d_1 \ln 2} > \frac{c_2}{2d_2 \ln 2}$. To derive the optimal price μ^* , we consider

the following three possible intervals.

- Case I-a: $\mu^* \in \left[\frac{c_2}{d_2 \ln 2}, \frac{c_1}{d_1 \ln 2} \right)$. In this case, based on (7), we have $x_1 = \frac{c_1}{\mu \ln 2} - d_1$ and $x_2 = 0$. As a result, P1 is reduced to a convex optimization problem, and it can be solved that $\mu^* = \frac{c_1}{(u_k + d_1) \ln 2}$. Using the same approach as [33], it can be shown that μ^* is the optimal solution for P1 if and only if $\frac{c_2}{d_2 \ln 2} \leq \frac{c_1}{(u_k + d_1) \ln 2} < \frac{c_1}{d_1 \ln 2}$, i.e., $\frac{c_1}{c_2/d_2} - d_1 \geq u_k > 0$. Thus, it follows that

$$\mu^* = \frac{c_1}{(u_k + d_1) \ln 2}, \text{ if } \frac{c_1}{c_2/d_2} - d_1 \geq u_k > 0. \quad (16)$$

- Case I-b: $\mu^* \in \left[\frac{c_1}{2d_1 \ln 2}, \frac{c_2}{d_2 \ln 2} \right)$. In this case, based on (7), we have $x_1 = \frac{c_1}{\mu \ln 2} - d_1$ and $x_2 = \frac{c_2}{\mu \ln 2} - d_2$. As a result, P1 becomes a convex optimization problem, and it can be solved that $\mu^* = \frac{c_1 + c_2}{(u_k + d_1 + d_2) \ln 2}$. Same as Case I-a, it can be shown that the obtained μ^* is the optimal solution for P1 if and only if $\frac{c_1}{2d_1 \ln 2} \leq \frac{c_1 + c_2}{(u_k + d_1 + d_2) \ln 2} < \frac{c_2}{d_2 \ln 2}$, i.e., $\frac{c_1 + c_2}{c_1/2d_1} - d_1 - d_2 \geq u_k > \frac{c_1 + c_2}{c_2/d_2} - d_1 - d_2$. Thus, it follows that

$$\begin{aligned} \mu^* &= \frac{c_1 + c_2}{(u_k + d_1 + d_2) \ln 2}, \\ &\text{if } \frac{c_2}{c_1/2d_1} + d_1 - d_2 \geq u_k > \frac{c_1}{c_2/d_2} - d_1. \end{aligned} \quad (17)$$

- Case I-c: $\mu^* \in \left[\frac{c_2}{2d_2 \ln 2}, \frac{c_1}{2d_1 \ln 2} \right)$. In this case, based on (7), we have $x_1 = d_1$ and $x_2 = \frac{c_2}{\mu \ln 2} - d_2$. Same as the previous two subcases, P1 is reduced to a convex problem, and it follows that $\mu^* = \frac{c_2}{(u_k - d_1 + d_2) \ln 2}$. It is the optimal solution for P1 if and only if $\frac{c_2}{2d_2 \ln 2} \leq \frac{c_2}{(u_k - d_1 + d_2) \ln 2} < \frac{c_1}{2d_1 \ln 2}$, i.e., $d_1 + d_2 \geq u_k > \frac{c_2}{c_1/2d_1} + d_1 - d_2$. Thus, it follows that

$$\begin{aligned} \mu^* &= \frac{c_2}{(u_k - d_1 + d_2) \ln 2}, \\ &\text{if } d_1 + d_2 \geq u_k > \frac{c_2}{c_1/2d_1} + d_1 - d_2. \end{aligned} \quad (18)$$

Based on the above results, the optimal pricing strategy for the uploader under Case I can be summarized as

$$\mu^* = \begin{cases} \frac{c_1}{(u_k + d_1) \ln 2}, & \text{if } \frac{c_1}{c_2/d_2} - d_1 \geq u_k > 0, \\ \frac{c_1 + c_2}{(u_k + d_1 + d_2) \ln 2}, & \text{if } \frac{c_2}{c_1/2d_1} + d_1 - d_2 \geq u_k > \frac{c_1}{c_2/d_2} - d_1, \\ \frac{c_2}{(u_k - d_1 + d_2) \ln 2}, & \text{if } d_1 + d_2 \geq u_k > \frac{c_2}{c_1/2d_1} + d_1 - d_2. \end{cases} \quad (19)$$

Remark: It is observed from (19) that the optimal price can be divided into three regions based on the uploader's available bandwidth. Based on the demand of the peers and the supply

of the uploader, the three regions are named as *insufficient region*, *balance region*, and *sufficient region*. In the insufficient region, the uploader's bandwidth is not enough to support all the peers. In this region, at least one peer will be not assigned any bandwidth. Peers are excluded from the game based on their $\frac{c_i}{d_i}$ values. Peers with low values are rejected first. For instance, in Case I-a, peer 2 is rejected from the game and only peer 1 remains in the game. In the balance region, the uploader will allocate bandwidth to each peer. However, none of the peers can download at its maximum bandwidth d_i . In this region, the uploader allocates its limited bandwidth to the peers proportional to their $\frac{c_i}{d_i}$ values. In the sufficient region, the uploader's bandwidth is able to support both the peers, and at least one of them can download at its maximum download bandwidth. The peer with the largest $\frac{c_i}{d_i}$ will be the first peer that can download at its maximum download bandwidth. When the bandwidth is sufficiently large, both peers can download at their maximum download bandwidth. ■

Now, we consider Case II: $\frac{c_1}{d_1 \ln 2} > \frac{c_1}{2d_1 \ln 2} > \frac{c_2}{d_2 \ln 2} > \frac{c_2}{2d_2 \ln 2}$. Similar as Case I, we consider different intervals to find the optimal price μ^* in each interval.

- Case II-a: $\mu^* \in \left[\frac{c_1}{2d_1 \ln 2}, \frac{c_1}{d_1 \ln 2} \right)$. In this case, based on (7), we have $x_1 = \frac{c_1}{\mu \ln 2} - d_1$ and $x_2 = 0$. Same as Case I-a, P1 becomes a convex optimization problem, and it can be solved that $\mu^* = \frac{c_1}{(u_k + d_1) \ln 2}$. Using the same approach as [33], it can be shown that μ^* is the optimal solution for P1 if and only if $\frac{c_1}{2d_1 \ln 2} \leq \frac{c_1}{(u_k + d_1) \ln 2} < \frac{c_1}{d_1 \ln 2}$, i.e., $d_1 \geq u_k > 0$. Thus, it follows that

$$\mu^* = \frac{c_1}{(u_k + d_1) \ln 2}, \text{ if } d_1 \geq u_k > 0. \quad (20)$$

- Case II-b: $\mu^* \in \left[\frac{c_2}{d_2 \ln 2}, \frac{c_1}{2d_1 \ln 2} \right)$. In this case, we show that the maximum possible utility for the uploader is lower than that obtained in Case II-a, and hence the optimal price will never lie in this range. Based on (7), we have $x_1 = d_1$ and $x_2 = 0$. P1 is thus reduced to finding the maximum value of μd_1 , and is valid only when $d_1 \leq u_k$. Since $\mu \in \left[\frac{c_2}{d_2 \ln 2}, \frac{c_1}{2d_1 \ln 2} \right)$, the upper bound of μd_1 is $\frac{c_1}{2 \ln 2}$. However, when $d_1 \leq u_k$, it is observed from Case II-a that the maximum revenue is $\mu^* \left(\frac{c_1}{\mu^* \ln 2} - d_1 \right)$, where μ^* is given by (20). Thus, $\mu^* \left(\frac{c_1}{\mu^* \ln 2} - d_1 \right)$ can be computed as $\frac{c_1 u_k}{(u_k + d_1) \ln 2}$, which is larger than $\frac{c_1}{2 \ln 2}$ when $d_1 \leq u_k$. Thus, μ^* should not lie in this range.
- Case II-c: $\mu^* \in \left[\frac{c_2}{2d_2 \ln 2}, \frac{c_2}{d_2 \ln 2} \right)$. In this case, based on (7), we have $x_1 = d_1$ and $x_2 = \frac{c_2}{\mu \ln 2} - d_2$. It follows that $\mu^* = \frac{c_2}{(u_k - d_1 + d_2) \ln 2}$. It is the optimal solution for P1 if and only

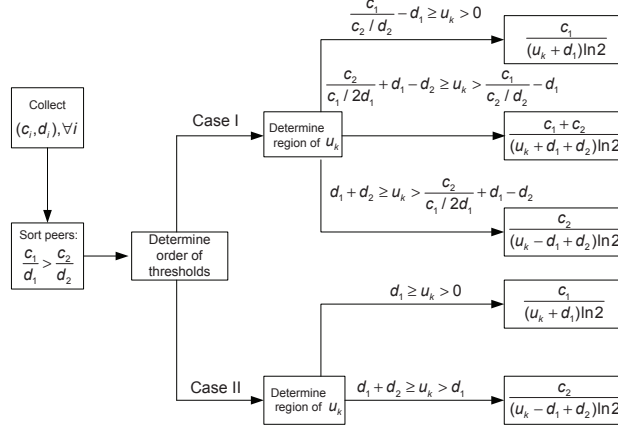


Fig. 2. Procedure of finding the optimal price for the two-peer scenario

if $\frac{c_2}{2d_2 \ln 2} \leq \frac{c_2}{(u_k - d_1 + d_2) \ln 2} < \frac{c_2}{d_2 \ln 2}$, i.e., $d_1 + d_2 \geq u_k > d_1$.

$$\mu^* = \frac{c_2}{(u_k - d_1 + d_2) \ln 2}, \text{ if } d_1 + d_2 \geq u_k > d_1. \quad (21)$$

Based on the above results, the optimal pricing strategy for the uploader under Case II can be summarized as

$$\mu^* = \begin{cases} \frac{c_1}{(u_k + d_1) \ln 2}, & \text{if } d_1 \geq u_k > 0, \\ \frac{c_2}{(u_k - d_1 + d_2) \ln 2}, & \text{if } d_1 + d_2 \geq u_k > d_1. \end{cases} \quad (22)$$

Remark: It is observed from (22) that the optimal price obtained under Case II can be divided into two regions based on the uploader's available bandwidth. We refer to these two regions as *insufficient region* and *sufficient region*. In the insufficient region, the uploader will only accept the request from peer 1, which is the peer with high $\frac{c_i}{d_i}$ value. In the sufficient region, the uploader will allocate peer 1 its maximum download bandwidth. It is observed that the price strategy will allocate bandwidth to peer 2 only when peer 1 is allocated its full download bandwidth d_1 . This is quite different from the scenario in Case I. This phenomenon happens due to the fact that the peer 1's $\frac{c_i}{d_i}$ value is much larger than that of peer 2. ■

In summary, the procedure to find the optimal price for the two-peer scenario is given in Fig. 2. It is not difficult to observe that the optimal price is determined by the following two factors:

- The order of the downloaders' thresholds.
- The uploader's available upload bandwidth.

Once these two factors are determined, the optimal price can be easily obtained. From an economic perspective, the order of the downloaders' thresholds actually reflect the demand and the purchasing power (i.e., the available accumulated credits) of the downloaders. The uploader's upload bandwidth reflects the market supply. The price of the goods is determined by the relationship between the supply and demand.

Another key result observed from the above solutions is that the optimal price μ^* is always obtained when (15) holds with equality, i.e., $\sum_{i \in S_k} f_i(\mu^*) = u_k$. This observation is very important, and plays a significant role in determining the Stackelberg equilibrium of the proposed game, which will be discussed later in Subsection 5.3.

2) *Multi-peer Scenario*: For the multi-peer scenario, there are more cases, and the number of cases increases with the increase of the number of peers. Thus, in general, we are not able to obtain a closed-form solution for the multi-peer scenario. However, once the order of the peers' thresholds is determined, a closed-form solution can be obtained. For the purpose of illustration, we derive the closed-form solution for P1 when the thresholds of the peers satisfy the following order $\frac{c_1}{d_1} > \dots > \frac{c_{|S_k|}}{d_{|S_k|}} > \frac{c_1}{2d_1} > \dots > \frac{c_{|S_k|}}{2d_{|S_k|}}$, where $|\cdot|$ denotes the cardinality of a set.

To avoid trivial solutions, we assume that $\sum_{i \in S_k} d_i > u_k$ in the following analysis. Due to the complexity of $f_i(\mu)$, P1 is difficult to solve directly. Therefore, to solve P1, we first consider the following problem

$$\text{P2: } \max_{\mu > 0} \sum_{i \in S_k} \mu \left(\frac{c_i}{\mu \ln 2} - d_i \right), \quad (23)$$

$$\text{s.t. } \sum_{i \in S_k} \left(\frac{c_i}{\mu \ln 2} - d_i \right) \leq u_k. \quad (24)$$

This problem is a convex optimization problem. Therefore, this problem can be solved by standard convex optimization techniques. Details are omitted here for brevity. The optimal price μ for P2 can be obtained as follows,

$$\mu = \frac{\sum_{i \in S_k} c_i}{(u_k + \sum_{i \in S_k} d_i) \ln 2}. \quad (25)$$

Now, we relate the optimal solution of P2 to that of P1 in the following proposition.

Proposition 4.3: The price μ given in (25) is the optimal solution of P1 if and only if

$$\frac{\sum_{i \in S_k} c_i}{\min_i \frac{c_i}{d_i}} - \sum_{i \in S_k} d_i < u_k < \frac{2 \sum_{i \in S_k} c_i}{\max_i \frac{c_i}{d_i}} - \sum_{i \in S_k} d_i, \quad (26)$$

when $\min_i \frac{c_i}{d_i} \geq \max_i \frac{c_i}{2d_i}$.

Proof: This proof consists of the following two parts.

Part 1: Sufficiency. The optimal price μ^* given by (25) is the optimal solution of P1 if $\frac{c_i}{2d_i \ln 2} \leq \mu < \frac{c_i}{d_i \ln 2}, \forall i$. Substituting (25) into these inequalities yields $\frac{\sum_{i \in S_k} c_i}{\frac{c_i}{d_i}} - \sum_{i \in S_k} d_i < u_k \leq \frac{2 \sum_{i \in S_k} c_i}{\frac{c_i}{d_i}} - \sum_{i \in S_k} d_i, \forall i$. Thus, when $\min_i \frac{c_i}{d_i} \geq \max_i \frac{c_i}{2d_i}$, these inequalities can be compactly written as

$$\frac{\sum_{i \in S_k} c_i}{\min_i \frac{c_i}{d_i}} - \sum_{i \in S_k} d_i < u_k \leq \frac{2 \sum_{i \in S_k} c_i}{\max_i \frac{c_i}{d_i}} - \sum_{i \in S_k} d_i. \quad (27)$$

The “if” part is thus proved. Next, we consider the “only if” part, which is proved by contradiction as follows.

Part 2: Necessity. For the ease of exposition, we assume that the peers are sorted by the following order: $\frac{c_1}{d_1} > \dots > \frac{c_{|S_k|}}{d_{|S_k|}} > \frac{c_1}{2d_1} > \dots > \frac{c_{|S_k|}}{2d_{|S_k|}}$. In order to prove the necessity, we suppose that the price μ^* given in (25) is optimal even if the inequality given in (26) does not hold. We consider on possible region for μ^* below, and a similar proof applies to the other regions. Suppose u_k satisfies the following inequality

$$\frac{\sum_{i=1}^{|S_k|-1} c_i}{\frac{c_{|S_k|-1}}{d_{|S_k|-1}}} - \sum_{i=1}^{|S_k|-1} d_i < u_k \leq \frac{\sum_{i=1}^{|S_k|} c_i}{\frac{c_{|S_k|}}{d_{|S_k|}}} - \sum_{i=1}^{|S_k|} d_i, \quad (28)$$

and μ^* given by (25) is still optimal when (28) holds. Since $u_k \leq \sum_{i=1}^{|S_k|} c_i / \frac{c_{|S_k|}}{d_{|S_k|}} - \sum_{i=1}^{|S_k|} d_i$, it follows that $\frac{c_{|S_k|}}{d_{|S_k|}} \leq \frac{\sum_{i=1}^{|S_k|} c_i}{u_k + \sum_{i=1}^{|S_k|} d_i}$. Then, according to (25), we have that $\mu^* \geq \frac{c_{|S_k|}}{d_{|S_k|}}$. Then, it follows from (7) that $x_{|S_k|} = 0$. This indicates that the peer with the smallest $\frac{c_i}{d_i}$ will be excluded from the game under the above condition.

Then, it follows that μ^* must be the optimal solution of P1 with $|S_k| - 1$ peers, which is given as follows

$$\max_{\mu > 0} \sum_{i=1}^{|S_k|-1} \mu f_i(\mu), \quad \text{s.t.} \quad \sum_{i=1}^{|S_k|-1} f_i(\mu) \leq u_k. \quad (29)$$

Thus, under the condition given by (28), using the same way as the proof of the previous “if” part, it can be shown that the optimal solution for this problem is given by

$$\tilde{\mu}^* = \frac{\sum_{i=1}^{|S_k|-1} c_i}{\left(u_k + \sum_{i=1}^{|S_k|-1} d_i\right) \ln 2}. \quad (30)$$

It is easy to observe that the optimal price $\tilde{\mu}^*$ given in (30) for the above problem is different from μ^* given by (25). Thus, this contradicts with our presumption that μ^* is optimal for P1 with u_k satisfying (28). Using the same method, we can prove that μ^* is not the optimal for P1 for other regions. Therefore, the interference vector μ^* given by (25) is the optimal solution of P1 only if u_k satisfying (27). The “only if” part thus follows.

By combining the proofs of both the “sufficiency” and “necessity” parts, Proposition 4.3 is thus proved. ■

With the results obtained above, we can solve a series of similar sub-problems of P1. Then, combining these obtained results by the same approach as the two-peer scenario, we can obtain the following theorem.

Theorem 4.2: When the thresholds of the peers satisfy $h_1 > \dots > h_{|S_k|} > h_1/2 > \dots > h_{|S_k|}/2$, where $h_i \triangleq c_i/d_i$, the optimal price μ^* for P1 is then given by

$$\mu^* = \begin{cases} p_{|S_k|}, & \text{if } R_{|S_k|} \geq u_k > R_{|S_k|-1} \\ \vdots & \vdots \\ p_2, & \text{if } R_2 \geq u_k > R_1 \\ q_{|S_k|}, & \text{if } R_1 \geq u_k > T_{|S_k|} \\ q_{|S_k|-1}, & \text{if } T_{|S_k|} \geq u_k > T_{|S_k|-1} \\ \vdots & \vdots \\ q_1, & \text{if } T_2 \geq u_k > T_1 \end{cases}, \quad (31)$$

where $q_K = \frac{\sum_{i=1}^K c_i}{(u_k + \sum_{i=1}^K d_i) \ln 2}$, $T_K = \frac{\sum_{i=1}^K c_i}{h_K} - \sum_{i=1}^K d_i$, $p_K = \frac{\sum_{i=K}^{|S_k|} c_i}{(u_k - \sum_{i=1}^{K-1} d_i + \sum_{i=K}^{|S_k|} d_i) \ln 2}$ and $R_K = \frac{2 \sum_{i=K}^{|S_k|} c_i}{h_K} + \sum_{i=1}^{K-1} d_i - \sum_{i=K}^{|S_k|} d_i$.

For other cases of the multi-peer scenario, closed-form solutions can also be obtained in the same way. In general, the optimal pricing strategy for the multi-peer scenario can be obtained by the same procedure illustrated in Fig. 2. For the same type of peer (i.e., the same d), the optimal pricing scheme tends to allocate more bandwidth to peers with higher contribution (i.e., more credits c). This indicates that the obtained pricing strategy for the multi-peer scenario can provide a strong incentive for peers to cooperate with each other. It is also observed that the optimal price μ^* is always obtained when (15) holds with equality, i.e., $\sum_{i \in S_k} f_i(\mu^*) = u_k$.

C. Stackelberg Equilibrium of the Proposed Game

In this subsection, we investigate the SE for the proposed Stackelberg game, and show the SE is unique and Pareto-optimal when u_k is given.

With the optimal solution obtained in Theorem 4.1 and 4.2, the SE for the proposed Stackelberg game is given as follows.

Theorem 4.3: The SE for the Stackelberg game formulated by the uploading game and the downloading game is (\mathbf{x}^*, μ^*) , where \mathbf{x}^* is given by (7), and μ^* is the optimal solution of P1.

Proof: Since μ^* is the optimal solution of P1, we have $U^{up}(\mu^*, \mathbf{x}) \geq U^{up}(\mu, \mathbf{x})$ for any given vector \mathbf{x} . Thus, it follows that $U^{up}(\mu^*, \mathbf{x}^*) \geq U^{up}(\mu, \mathbf{x}^*)$, where \mathbf{x}^* is given by (7). Similarly, since \mathbf{x}^* is the optimal solution for the downloading game, we have $U_i^{down}(\mathbf{x}_i^*, \mu) \geq U_i^{down}(\mathbf{x}_i, \mu), \forall i$, for any given price μ . Thus, it follows that $U_i^{down}(\mathbf{x}_i^*, \mu^*) \geq U_i^{down}(\mathbf{x}_i, \mu^*), \forall i$. Then, combining the above two facts, according to the definition of SE given in Definition 3.1, (\mathbf{x}^*, μ^*) is the SE for the proposed Stackelberg game. ■

Now, we show that the SE is unique and Pareto-optimal when u_k is given.

Theorem 4.4: The SE for the proposed Stackelberg game is unique and Pareto-optimal for a given u_k .

Proof: First, we show that the SE for the proposed Stackelberg game is unique for a given u_k . As pointed out in the previous subsection, the optimal pricing strategy is unique when the order of peers' thresholds and u_k are given. The order of peers' thresholds is determined by the values of c_i and $d_i, \forall i$, which are fixed during each implementation of the Stackelberg game. Thus, it is clear that the optimal price μ^* is unique for a given u_k . On the other hand, it is observed from (7) that the download bandwidth for each peer is unique under a given μ . Thus, it is obvious that the SE for the proposed game is unique under a given u_k .

Now, we show that the SE is Pareto-optimal for a given u_k . Given an initial resource allocation scheme among a group of peers, a change to a different allocation scheme that makes at least one peer better off without making any other peers worse off is called a Pareto improvement. An allocation scheme is defined as "Pareto-optimal" when no further Pareto improvements can be made. In other words, in a Pareto-optimal equilibrium, no one can be made better off without making at least one individual worse off. It is observed that the optimal μ^* always satisfies $\sum_{i \in S_k} f_i(\mu^*) = u_k$. Thus, increasing one peer's (e.g., peer 1) bandwidth allocation will inevitably decrease another peer's (e.g. peer 2) bandwidth allocation. This makes peer 2's bandwidth

allocation deviates from its optimal bandwidth allocation, and consequentially decreasing its utility. Thus, no peer can be made better off without making some other peer worse off, and the SE is Pareto-optimal. ■

VI. IMPLEMENTATION OF THE STACKELBERG GAME IN P2P STREAMING NETWORKS

In previous section, we have solved the proposed Stackelberg game and obtained its SE. In this section, we investigate how to implement the proposed game in P2P networks in detail. Two implementation methods referred to as *direct implementation* and *bargaining implementation* are proposed and investigated as below.

A. Direct Implementation

Direct implementation is strictly based on the obtained results given in Section V. It is a one-round implementation with four stages, which are described as follows.

- **Stage 1:** All the peers requesting data from the uploader send their contribution values c_i and download bandwidth limits d_i to the uploader.
- **Stage 2:** Having received the data requests together with the information of c_i and d_i from all the peers, the uploader first sorts all the peer in the order $\frac{c_1}{d_1} > \dots > \frac{c_{|S_k|}}{d_{|S_k|}}$, and determines the order of peers' thresholds. Then, the uploader computes the optimal price μ^* using the same approach as illustrated in Fig. 2, and broadcasts the optimal price μ^* to all the peers.
- **Stage 3:** Based on the received price, each peer computes its optimal download bandwidth x_i^* based on (7), and sends the calculated results to the uploader.
- **Stage 4:** The uploader allocates the bandwidth based on $x_i^*, \forall i \in S_k$ and starts streaming.

B. Bargaining Implementation

In this subsection, we propose the bargaining implementation for the proposed Stackelberg game based on the characteristics of P1. It can be shown that P1 is equivalent to the following problem

$$\text{P3: } \max_{\mu > 0} \quad \mu, \tag{32}$$

$$\text{s.t. } \sum_{i \in S_k} f_i(\mu) = u_k. \tag{33}$$

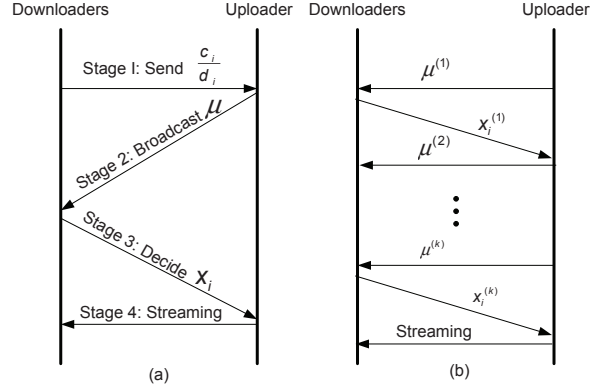


Fig. 3. Implementation in P2P Streaming Networks: (a) Direct implementation (b) Bargaining implementation

Then, we propose the bargaining implementation based on the following two facts: (i). $f_i(\mu)$ is a decreasing function of μ , which can be observed from (7). (ii). The upper limit of μ^* is $\max_i \frac{c_i}{d_i \ln 2}$. This can be proved using the same approach as Proposition 4.1.

- **Stage 1:** The uploader sets an initial price μ (where $\mu \geq \max_i \frac{c_i}{d_i \ln 2}$), and broadcasts it to all the downloaders.
- **Stage 2:** Each downloader computes its optimal download bandwidth x_i based on (7) for the given μ , and send back x_i to the uploader.
- **Stage 3:** Having received x_i from all the peers, the uploader computes the total demand $\sum_{i \in S_k} x_i$, and compares the total demand with its upload bandwidth u_k . Assume that ϵ is a small positive constant that controls the algorithm accuracy. If $\sum_{i \in S_k} x_i < u_k - \epsilon$, the uploader decreases the bandwidth price by $\Delta\mu$, where $\Delta\mu$ is a small step size. After that, the uploader broadcasts the new price to all the downloaders.
- **Stage 4:** Stage 2 and Stage 3 are repeated until $|\sum_{i \in S_k} x_i - u_k| < \epsilon$. Then, the uploader starts streaming.

The convergence of the bargaining algorithm is guaranteed by the following facts: (i). The optimal price μ^* is always obtained when the upload bandwidth of the uploader is fully allocated, i.e., $\sum_{i \in S_k} x_i = u_k$. (ii). $f_i(\mu)$ is a decreasing function of μ . (iii). The SE for the proposed Stackelberg game is unique and Pareto-optimal for a given u_k .

C. Direct Implementation Vs. Bargaining Implementation

In this subsection, we analyze and compare the difference between these two kinds of implementation schemes.

It is not difficult to observe that the direct implementation is time-saving, since it only needs one-round to determine the optimal bandwidth price and the optimal download bandwidth of the peers. In contrast, the bargaining implementation requires much more time. This is due to the fact that the uploader and the downloaders have to go through a multi-round bargaining process to finally reach the equilibrium. Thus, for delay-sensitive service, such as P2P multimedia streaming, direct implementation is preferred.

Another difference between these two implementation schemes is the requirement on the computing power of the uploader. It is observed that direct implementation requires the uploader to directly compute the optimal price based on the procedure given in Section V-B, which is a complex procedure involving a lot of cases. Thus, it has a high requirement on the computing power of the uploader. In contrast, the bargaining implementation greatly relieves the computation burden on the uploader. The uploader only needs to compare the total demand with its upload bandwidth, which requires much less computing power. Thus, the bargaining implementation should be preferred by the handheld mobile devices with less computing power.

It is worthy pointing out that no matter which implementation scheme is employed, the same Stackelberg equilibrium results for the same set data. This is due to the fact that the SE is unique which is proved in Theorem 4.4.

VII. DEALING WITH DYNAMICS OF P2P STREAMING NETWORKS

P2P networks are dynamic in nature. Peers may leave or join the network at any time. How the equilibrium changes when peers leave or join the network is of great importance to the study of a dynamic network. Thus, in this subsection, we investigate whether the equilibrium will change and how it will change under these situations.

When a peer joins the network, it is given a certain number of credits. The initial credits for each peer can be the same (e.g., 100 credits for each peer) or different (e.g., c_i for peer i). The credits of a peer is updated after each transaction. One transaction means that a downloading peer finished its downloading from a uploader. After one transaction of downloader i , its credits c_i is updated by $c_i = c_i - \mu * x_i$, and the credits of the uploader j is updated by $c_j = c_j + \mu * x_i$. If

multiple downloader finish their downloading at the same time, the uploader updates its credits by collecting credits from them together.

To facilitate the analysis, we assume that there are N downloading peers and 1 uploading peer at the original SE. The original SE is denoted by (μ^*, \mathbf{x}^*) , where \mathbf{x}^* is the optimal bandwidth allocation vector for the downloading peers at the SE. The new SE after peers leaving or joining the network is denoted by $(\tilde{\mu}^*, \tilde{\mathbf{x}}^*)$. Besides, we assume that the information that a downloader leaves or joins the network is only available to the uploader itself, and it will not share the information with other downloaders.

A. Peers Leaving the P2P Streaming Network

When a peer j leaves a P2P streaming network, the SE changes only when

$$U^{up}(\tilde{\mu}^*, \tilde{\mathbf{x}}^*) \geq U^{up}(\mu^*, \mathbf{x}^*) - U^{up}(\mu^*, x_j), \quad (34)$$

where $U^{up}(\tilde{\mu}^*, \tilde{\mathbf{x}}^*)$ denotes the utility of the uploader at the new SE, $U^{up}(\mu^*, \mathbf{x}^*)$ denotes the utility of the uploader at the original SE, and $U^{up}(\mu^*, x_j)$ denotes peer j 's contribution to the uploader's utility at the original SE.

For the problem considered in this paper, the inequality (34) always holds. Thus, when a downloading peer leaves the network, the best strategy is to re-implement the Stackelberg game with the remaining $N - 1$ peers.

B. Peers Joining the P2P Streaming Network

When a peer joins the network, the SE changes only when

$$U^{up}(\tilde{\mu}^*, \tilde{\mathbf{x}}^*) > U^{up}(\mu^*, \mathbf{x}^*). \quad (35)$$

When a peer joins the network, the number of competing peers increases. As the competition between downloading peers becomes fiercer, the uploading peer has the incentive to increase the price of the resource to increase its revenue. It is worth pointing out that (35) does not always hold. For example, if the c_i/d_i value of the joining peer is very small, this peer will be rejected, and the SE will be sustained.

A simple way to re-attain the equilibrium is to completely re-implement the Stackelberg game again with the $N + 1$ peers. This method is guaranteed to reach a new SE which is unique and

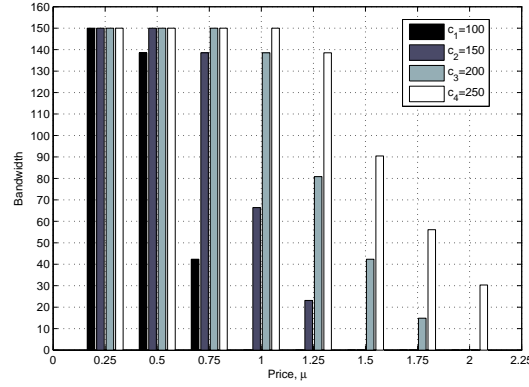


Fig. 4. Bandwidth allocation vs. Price under same connection types

Pareto-optimal. From the uploader's perspective, this method is beneficial since the new SE will never decrease its utility. The uploader need only recompute the price μ^* of the resource, taking the new peer's c_i and d_i into consideration, and broadcasts the calculated μ_{new}^* to all the $N+1$ peers. Though this method is good for the uploader, it sacrifices the existing downloaders' interests. This is due to the fact that $\sum_{i \in \mathcal{S}_k} x_i = u_k$ always holds at the equilibrium. If a new peer joins and is allocated a certain amount of download bandwidth, some of the existing peers' download bandwidth must decrease. For some peers, even though their download bandwidth may not decrease, their utility decreases due to the increase of the resource price.

VIII. PERFORMANCE EVALUATION

In this section, several numerical examples are provided to evaluate the performance of the proposed incentive resource allocation scheme. It is shown that the proposed resource allocation scheme can provide strong incentives for peers to contribute to the P2P network.

A. Example 1: Peers with the same connection type but different contribution values

In this example, we assume that there are four peers requesting data chunks from the uploader k . The connection types of all the requesting peers are assumed to be the same, and their maximum download bandwidth are $d_1 = d_2 = d_3 = d_4 = 150$. The contribution values of these requesting peers at the current time are $[c_1, c_2, c_3, c_4] = [100, 150, 200, 250]$. The bandwidth assignments for the four downloading peers under different prices are shown in Fig. 4.

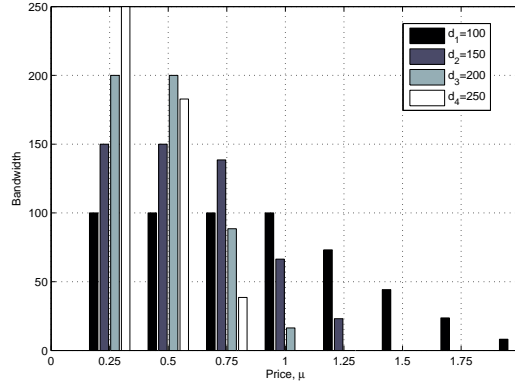


Fig. 5. Bandwidth allocation vs. Price under same contribution values

It is observed from Fig. 4 that the bandwidth assigned to each peer decreases with the increase of the price. For the same price, although all the peers have the same connection type, peers with larger contribution values are assigned higher bandwidth under our resource allocation strategy. This illustrates that the proposed resource allocation strategy can provide differentiated service to peers with different contribution, and thus encourage peers to contribute to the network.

B. Example 2: Peers with the same contribution value but different connection types

In this example, we assume that there are four peers requesting data chunks from the uploader k . We assume the contribution values of the requesting peers are the same, and are given by $[c_1, c_2, c_3, c_4] = [150, 150, 150, 150]$. The connection types of the requesting peers are assumed to be different, and are given by $[d_1, d_2, d_3, d_4] = [100, 150, 200, 250]$. The bandwidth assignments for this setup are shown in Fig. 5.

It is observed from Fig. 5 that when the price is low, every peer can download at its maximum download bandwidth. The bandwidth assigned to each peer decreases with the increase of the price. It is also observed that our resource allocation scheme biases toward peers with smaller download capacities. This is as expected. Intuitively, given the same unit of bandwidth resource, a peer with a smaller download capacity achieves a higher performance satisfaction factor than a peer with a larger download capacity.

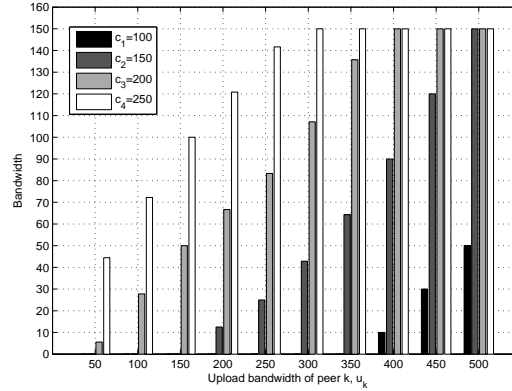


Fig. 6. Bandwidth allocation vs. u_k

C. Example 3: Relationship between bandwidth allocation and the available upload bandwidth

In this example, we assume that there are four peers requesting data chunks from the uploader k . The connection types of all the requesting peers are assumed to be the same, and their maximum download bandwidths are $d_1 = d_2 = d_3 = d_4 = 150$. The contribution values of these requesting peers at the current time are $[c_1, c_2, c_3, c_4] = [100, 150, 200, 250]$. The bandwidth assignments for the four downloading peers under u_k are shown in Fig. 6. It is observed from Fig. 6 that our resource allocation scheme gives a higher priority to peers with higher contribution in bandwidth assignment. When the available upload bandwidth u_k is small, the uploader will reject the request from the peers with low contribution, and provide the limited resource to the peers with high contribution. When the available upload bandwidth u_k is large, the uploader will try to meet every peer's request. However, peers with higher contribution values are given a higher priority in obtaining the bandwidth. It is also observed that with the increasing of u_k , the bandwidth assigned for each peer increases. This is due to the fact that the uploader's utility is maximized only when it contributes all its available upload bandwidth.

D. Example 4: Join of Competing Peers

In this example, for the purpose of comparison, we use the same system setup and simulation parameters as [29]. We assume the uploader's available bandwidth u_k is 2 Mb/s. There are four competing peers requesting data chunks from the uploader k . The connection types of the requesting peers are assumed to be different, and are given by $[d_1, d_2, d_3, d_4] = [2, 1.5, 1, 0.5]$ (in

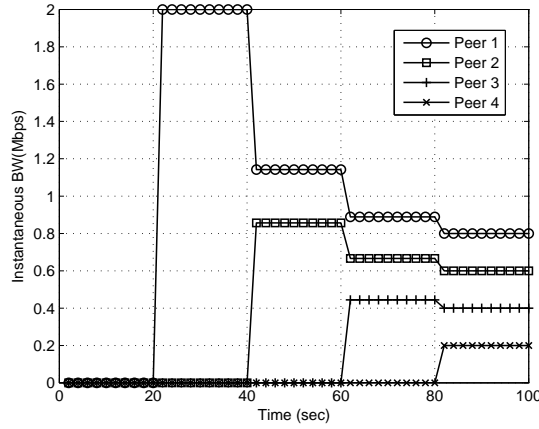


Fig. 7. Instantaneous Bandwidth Allocations: Join of Competing Peers

Mb/s). The arrival times of these peers are $t = 20, 40, 60$ and 80 s, respectively. The contribution values of these requesting peers are assumed to be $[c_1, c_2, c_3, c_4] = [400, 300, 200, 100]$. The results are obtained using the bargaining implementation. The initial value for μ is $\mu^{(0)} = \max_i \frac{c_i}{d_i \ln 2} = 288.539$. The step size for μ is chosen as 0.01 , and ϵ is chosen as 0.001 .

It is observed from Fig. 7 that the equilibrium of the game changes whenever a new competing peer joins the game. The bandwidth allocation for the existing peers decrease due to the newcomer. This is in accordance with our analysis given in Section VII-B. It is also observed that the uploader assigns all its bandwidth without reservation at each new equilibrium. Besides, at each equilibrium, the bandwidth allocation is proportional to the contribution value of each peer. This indicates that the proposed incentive mechanism is adaptive to the dynamics of the P2P network, and can always provide differentiated service to peers with different contribution values. It is also observed that the proposed scheme can achieve the same performance as that of [29].

E. Example 5: Leave of Competing Peers

In this example, we consider an opposite scenario of Example 4. We consider the scenario that peers leave the system one by one. For the convenience of analysis, we use the same system setup and simulation parameters as example 4. We assume that the four peers join the network at $t = 20$ s, and they leave the network one by one. The leave times of peer 4, 3, 2 are

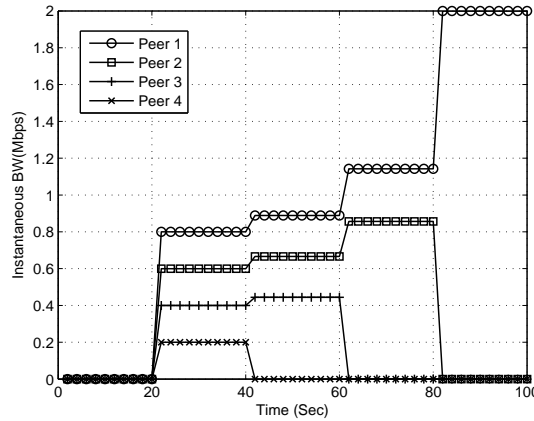


Fig. 8. Instantaneous Bandwidth Allocations: Leave of Competing Peers

$t = 40, 60, 80$ s, respectively.

From Fig. 8, it is observed that the equilibrium of the game changes whenever a competing peer leaves the network. The bandwidth allocation for the existing peers increase due to the leave of peers. This is in accordance with our analysis given in Section VII-A. It is also observed that the bandwidth assignment is proportional to the contribution value of each peer at each equilibrium. This indicates that the proposed incentive mechanism is robust to the dynamics of the P2P network, and can always provide differentiated service to peers with different contribution values. It is also interesting to observe that the equilibriums of Example 5 are exactly the same as those of the Example 4 for the same number of peers. This is due to fact that the uploader's utility obtained by selling the remaining bandwidth to the remaining peers is larger than that obtained by maintaining the current status in this example.

IX. DISCUSSIONS AND FUTURE WORK

A. Competition among Multiple Uploaders

In this paper, we consider a simple model where there is one uploader and multiple downloaders. In reality, there may exist multiple uploaders having overlapping data chunks. This implies that there may exist a competition among these uploaders, which may affect the pricing strategies of the uploaders. Our incentive mechanism can be applied to this scenario with few modifications.

The presence of multiple uploaders induces a subgame that involves the peers choosing the uploaders. This adds an additional "Step 0: Peers choose their uploader." to the proposed algorithms. Given any set of choices by the peers, a Stackelberg game is induced at each uploader, which can be solved for a unique SE according to the analysis done in previous sections. Thus, the key issue is how to choose the uploaders. When a new peer joins the network, it has to choose one uploader from multiple uploaders that have its desired data chunks. The prices at these uploaders observed by the newcomer at this moment are fixed. Thus, it is reasonable for the newcomer to choose the uploader with the lowest price.

However, it is worth pointing out that this scheme is in general suboptimal. This is due to the fact that the price at the new SE with the newcomer may not be the same as the price at the old SE without the newcomer. Thus, it is possible that the newcomer unilaterally deviate in its choice of the uploader to achieve a higher utility. If we take this into consideration, the game will become very complex and highly difficult to analyze. Thus, we would like to delegate this to our future work.

B. Trust Issues

In this paper, we focus on designing an incentive mechanism for P2P networks. However, it is worth pointing out that trust issues are also very important for P2P systems. For example, the proposed algorithms need the downloading peers to report their c_i 's and d_i 's to the uploader. Malicious peers may misreport their credits c_i 's and their types d_i 's to gain advantages against other peers. For example, a malicious peer may deliberately reports a bandwidth \tilde{d}_i smaller than its real demand bandwidth d_i to increase its priority (c_i/d_i) in obtaining bandwidth.

Another security issue is that malicious peers may deliberately upload polluted data chunks to other peers. Without effective measures to identify malicious peers, the polluted data chunks could be disseminated to the whole network more quickly in a P2P network with incentive mechanisms than that without incentive mechanisms. This is due to the fact that peers are motivated to upload data chunks to each other to earn points or monetary rewards in a P2P system with incentive mechanisms. Without the ability to identify malicious peers, peers are more likely to forward polluted data chunks, consequently degrading the performance of the system.

To deal with these trust issues, trust management schemes are needed to identify and defend

against malicious peers. In other words, incentive mechanisms must be used in trusted environments or together with reliable trust management mechanisms. Though trust management for P2P networks has been extensively studied in literature [34]–[40], joint design of trust management and incentive mechanisms for P2P networks remains unstudied. Due to the complexity and the lack of space, we leave this as our future work.

X. CONCLUSION

In this paper, a credit-based incentive mechanism to stimulate the cooperation between peers in a P2P streaming network is proposed. Taking the peers' heterogeneity and selfish nature into consideration, a Stackelberg game is designed to provide incentives and service differentiation for peers with different credits and connection types. The optimal pricing and purchasing strategies, which can jointly maximize the uploader's and the downloaders' utility functions, are derived by solving the Stackelberg game. The Stackelberg equilibrium is shown to be unique and Pareto-optimal. Then, two fully distributed implementation schemes are proposed and studied. It is shown that each of these schemes has its own advantages. The impact of peer churn on the proposed incentive mechanism is then analyzed. It is shown that the proposed mechanism can adapt to dynamic events such as peers joining or leaving the network. Finally, several numerical examples are presented, which show that the proposed incentive mechanism is effective in encouraging peers to cooperate with each other.

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